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PRELIMINARY RESULTS CONCERNING THE IMPROVEMENTS REALIZABLE THROUGH THE USE OF VARIABLE THRUST TOGETHER WITH ENGINE GIMBALING FOR A PARTICULAR INTERCEPTOR MISSILE

by

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ABSTRACT:

Some interceptor missiles as presently formulated possess a programmed thrust magnitude history with a gimbaled engine to provide steering. We examine one such missile to determine whether performance can be improved if we allow a variable thrust magnitude together with engine gimbaling to provide control.

Two trajectory optimization programs were written to provide an initial answer to this problem. Preliminary results indicate reductions in the time to intercept by as much as thirty per-cent over that obtained by the presently used guidance scheme. With tuning of the programs it seems reasonable to expect even greater improvements and further investigation seems warranted.

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Introduction

Some interceptor type missiles as presently formulated possess programmed thrust magnitude history with a gimbaled engine to provide steering. The present guidance scheme used on these missiles determines the steering control and hence the direction of the thrust vector. We examine one such missile and answer the question as to whether performance can be improved if we allow a variable thrust magnitude together with thrust direction to be controlled by some guidance scheme.

In order to take the first step in answering this question, two trajectory optimization programs were written. These were designed to determine optimal histories of thrust magnitude and direction in order to obtain minimum time to interception for our missile under given scenarios. While the programs are not in a finely tuned state, nevertheless, preliminary results indicate reductions in the time to intercept by as much as thirty per cent from that obtained by the present scheme. With tuning of the programs it seems reasonable to expect even greater improvements and further investigation seems warranted.

Model

The missile model used was two dimensional since all test trajectories were flown in a horizontal plane.

Letting the indicated terms have the meaning specified in the nomenclature, then the picture of the model is:

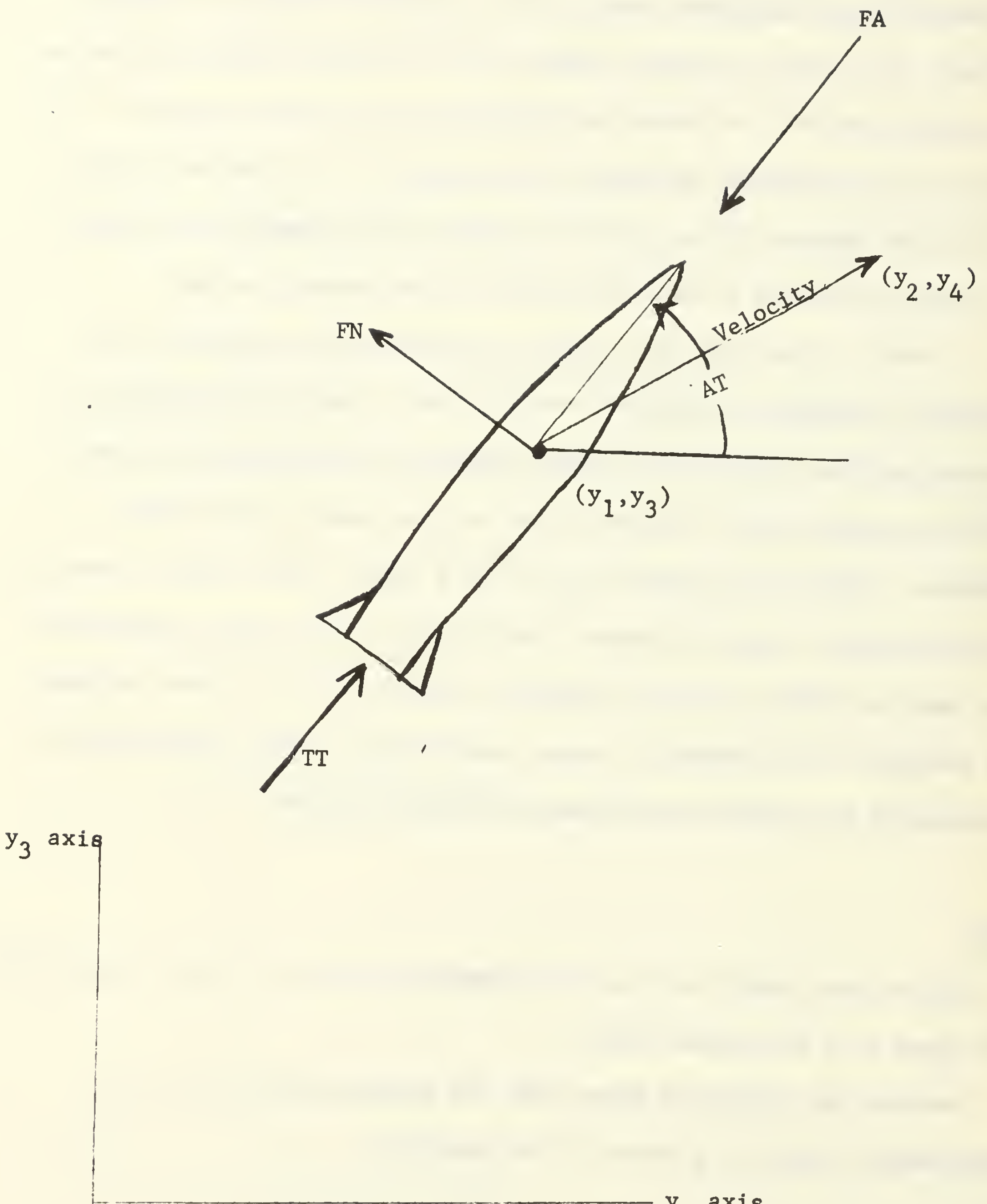


Figure 1

Missile Model

The differential equations for this model are[1] :

1a) $\dot{y}_1 = y_2$

1b) $\dot{y}_2 = \frac{TT-FA}{y_5} \cos AT - \frac{FN}{y_5} E_1$

1c) $\dot{y}_3 = y_4$

1d) $\dot{y}_4 = \frac{TT-FA}{y_5} \sin AT - \frac{FN}{y_5} E_2$

1e) $\dot{y}_5 = - \frac{TT}{8050}$

where i) $y_1,\ldots,y_5$ are called state variables since they define the state of the missile and TT,AT are called control variables since they control the state through the equations 1); ii) FA,FN are functions of the velocity vector and the control angle AT.

The constraints for this problem are

2a) $0 \leq TT \leq 14400.0$

2b) $\int_0^{TF} TTdt \leq 38{,}500$

in which 2a) is a thrust level constraint which says that our thrust must be non-negative and is bounded above by 14400 lbs. and 2b) is a condition on the amount of fuel used.

Our task is, given the initial conditions

3a) $y_{1_0},\ y_{2_0},\ y_{3_0},\ y_{4_0},\ y_{5_0}$

for the missile and

$$y_{1T_0},\ \dot{y}_{1T_0},\ y_{3T_0},\ \dot{y}_{3T_0}$$

for the target, then determine a history of TT, AT in time which

[1] Detailed equations are presented in the Appendix

yields a minimum for the time of intercept TF. Using the penalty method to include the constraint of target impact in the cost function, our cost function is then

4) $$c = TF + UN[(y_1 - y_{1T})^2 + (y_3 - y_{3T})^2]$$

Method of Solution

A. General Techniques Available

There are many ways to attack a problem of the type specified above. For example;

a) the classical calculus of variations technique

b) gradient technique

c) conjugate gradient technique

Of these a) is an indirect method, which seeks a trajectory which satisfies certain necessary conditions rather than seeking to reduce the cost function directly. This method depends upon the choice of the initial values of a set of multipliers called adjoint variables which satisfy a certain system of differential equations. This choice is often a highly sensitive one and instability in attempting to converge to a solution trajectory can result.

Methods b) and c) are direct methods in that they directly seek to minimize the cost function by seeking new trajectories with lower values of cost function. All of these methods are based on generating a sequence of trajectories which converges to the minimizing one. The gradient technique works by linearizing the cost function at each trajectory of the sequence developed and iterates to the next trajectory of the sequence by changing the controls in the direction opposite to the gradient. The conjugate gradient technique is a step more sophisticated than the gradient technique in that it generates new trajectories in its sequence by effectively expanding the cost function in a Taylor Series up through the second order, thus obtaining a more accurate representation of this function.

All of these methods together with a number of others were considered for the problem at hand and because of greater sureness of convergence the conjugate gradient method was selected.

B. <u>Brief Description of the Conjugate Gradient Technique</u>

This method is most easily described when discussing the problem of minimizing a cost function which is a quadratic function of the N variables $x_1, \ldots, x_n$. Thus assume that we are given the problem selecting values of $x_1, \ldots, x_n$ in order to obtain a minimum of the quadratic function

5) $$c(X) = d + BX + 1/2\ X^T GX$$

where: i) X denotes the vector $(x_1, \ldots, x_n)$; ii) d denotes a constant and B denotes a constant vector; iii) G denotes the matrix of second partial derivatives of c. Given a starting point X_0 the conjugate gradient method computes a sequence of vectors H_0, $H_1, \ldots,$ along which the function c is minimized. Thus starting at X_0 the method computes a direction H_0 which depends on the cost function c and the point X_0 and determines a value X_1 which is a minimum of c in that direction. Next, a direction H_1 is computed at X_1 and c is minimized along that direction to produce the point X_2. The sequence continues in this manner and it can be shown that in the absence of round-off, the method will converge to the minimum point in at most N iterations (where N is the dimension of the vector X).

In general, as in our case, the cost function is not quadratic. The procedure then is to approximate the cost function by the first three terms if its Taylor Series at each iteration point so that it has the form of a quadratic and to develop the directions H_i from those approximations as outlined above for the quadratic case. Details of the conjugate gradient method as originally developed by Hestenes for linear systems, are in [1] and its application to general functions is explained in [2]. Furthermore, the technique of conjugate gradients works on more general functions than functions of a finite number of variables and one may apply it with some modification to functions of an infinite number of variables (see [3]). Thus for a cost function which depends upon an infinite number of variables as our cost function which depends upon the value of TT and AT at each time point, one may use this technique to seek out those values which minimize it.

C. Application of the Conjugate Gradient Technique to Our Problem

In order to apply the conjugate gradient technique to our problem, two computer programs were written.

The first of these programs was written using the conjugate gradient technique for an infinite number of variables as referred to above. This program is listed in the Appendix B and was never fully checked out due to lack of time.

The second program was written using the conjugate gradient method for functions of a finite number of variables as outlined above. Now as previously stated, the cost function for our problem depends upon infinite dimensional controls, namely the magnitude TT and direction AT of the thrust vector at each time point. However in any computing machine procedure for integrating the differential equations for our problem, only values of the controls TT and AT at a finite number of time points are used. For example, in the simplest type of integration scheme, if the time interval is denoted by DT and t_o, t_1, t_2,...t_j... are the time points of the integration scheme then

$$
6) \qquad
\begin{aligned}
y(t_1) &= y(t_0) + \dot{y}(t_0)\cdot DT \\
y(t_2) &= y(t_1) + \dot{y}(t_1)\cdot DT \\
&\vdots \\
y(t_{j+1}) &= y(t_j) + \dot{y}(t_j)\cdot DT \\
&\vdots \\
y(TF) &= y(TF-DT) + \dot{y}(TF-DT)\cdot DT
\end{aligned}
$$

where $y,\dot{y}$ denote the state variable to be integrated and its derivative and TF denotes the final time. In this scheme only the values of TT and AT at the time points t_i affect the trajectory. Thus our cost function which depends upon y at the final time in turn also depends on the values of TT and AT only at these time points.

Thus, the computer really reduces the infinite dimensional problem to a finite dimensional one. Furthermore if we take this into account in formulating our model then our numerical optimization scheme which must abide by such shortcomings of the computer, will be surer of success.

This then is the technique used to adapt the finite dimensional conjugate gradient method to our problem. The integration scheme selected is the one used on already existing trajectory computer programs for the missile under consideration and is as follows:

$$
\begin{aligned}
y_1(t_{j+1}) &= y_1(t_j) + f_1(t_j)\cdot DT + f_2(t_j)\cdot\frac{DT^2}{2} \\
y_2(t_{j+1}) &= y_2(t_j) + f_2(t_j)\cdot DT \\
y_3(t_{j+1}) &= y_3(t_j) + f_3(t_j)\cdot DT + f_4(t_j)\frac{DT^2}{2} \qquad (7) \\
y_4(t_{j+1}) &= y_4(t_j) + f_4(t_j)\cdot DT \\
y_5(t_{j+1}) &= y_5(t_j) + f_5(t_j)\cdot DT
\end{aligned}
$$

where we have denoted by f_i $i = 1,\ldots,5$ the right hand sides of 1). This integration scheme essentially integrates the position components y_1 and y_3 by using the first two derivatives of position, while integrating the velocity components y_2, y_4 and the mass y_5 by using only the first derivatives of these quantities.

Besides computation of the cost function at each iteration point, the conjugate gradient method requires us also to compute the derivative of the cost function with respect to the control variables $TT(t_i)$, $AT(t_i)$. By the chain rule for differentiation, this requires that we first differentiate the cost with respect to the state variables at TF and then differentiate the state variables at TF with respect to the controls at the times t_j. The former derivatives are easily formed, however the latter derivatives are formed sequentially as follows: According to the integration scheme 7) forming the derivative of $y_i (i = 1,\ldots,5)$ at t_0 with respect to $AT(t_0)$ and $TT(t_0)$ yields

$$
8) \qquad \frac{\partial y_i(t_0)}{\partial AT(t_0)} = 0 \qquad \frac{\partial y_i(t_0)}{\partial TT(t_0)} = 0 \qquad i = 1,\ldots,5
$$

Forming the derivative of y_i at t_1 with respect to $AT(t_1)$ and $TT(t_1)$ yields

9a) $$\frac{\partial y_i(t_1)}{\partial AT(t_1)} = 0 \qquad \frac{\partial y_i(t_1)}{\partial TT(t_1)} = 0 \qquad i = 1,\ldots,5$$

and next, forming derivatives with respect to $AT(t_o)$ yields

$$\frac{\partial y_i(t_1)}{\partial AT(t_0)} = \frac{\partial y_i(t_0)}{\partial AT(t_0)} + \left[\sum_{j=1}^{5} \frac{\partial f_i(t_0)}{\partial y_j(t_0)} \frac{\partial y_j(t_0)}{\partial AT(t_o)} + \frac{\partial f_i(t_0)}{\partial AT(t_0)}\right] \cdot DT$$

$$= \frac{\partial f_i(t_0)}{\partial AT(t_0)} \cdot DT \quad i = 2,4,5$$

9b) $$\frac{\partial y_i(t_1)}{\partial AT(t_0)} = \frac{\partial y_i(t_0)}{\partial AT(t_0)} + \left[\sum_{j=1}^{5} \frac{\partial f_i(t_0)}{\partial y_j(t_0)} \frac{\partial y_j(t_0)}{\partial AT(t_0)} + \frac{\partial f_i(t_0)}{\partial AT(t_0)}\right] \cdot DT$$

$$+ \left[\sum_{j=1}^{5} \frac{\partial f_{i+1}(t_0)}{\partial y_j(t_0)} \frac{\partial y_j(t_0)}{\partial AT(t_0)} + \frac{\partial f_{i+1}(t_0)}{\partial AT(t_0)}\right] \cdot \frac{DT^2}{2}$$

$$= \frac{\partial f_i(t_0)}{\partial AT(t_0)} \cdot DT + \frac{\partial f_{i+1}(t_0)}{\partial AT(t_0)} \cdot \frac{DT^2}{2} \qquad i = 1,3$$

where the last equalities in 9b) result from 9a) and where $f_i(t_k)$ means the function f_i evaluated with arguments $y(t_k)$, $AT(t_k)$, $TT(t_k)$. Similar equations hold for the derivatives with respect to $TT(t_1)$ and $TT(t_0)$. Continuing in this fashion, then at time t_k we form the derivatives of $y_i(t_k)$ with respect to TT and AT at all time points up through t_k. Forming the derivatives with respect to AT at all such times, first we set, (as in 9) the derivative with respect to $AT(t_k)$

10a) $$\frac{\partial y_i(t_k)}{\partial AT(t_k)} = 0 \qquad i = 1,\ldots,5$$

while for the derivative with respect to AT at the immediately preceeding time point t_{k-1}

10b) $$\frac{\partial y_i(t_k)}{\partial AT(t_{k-1})} = \frac{\partial f_i(t_{k-1})}{\partial AT(t_{k-1})} \cdot DT \qquad i = 2,4,5$$

$$\frac{\partial y_i(t_k)}{\partial AT(t_{k-1})} = \frac{\partial f_i(t_{k-1})}{\partial AT(t_{k-1})} \cdot DT + \frac{\partial f_{i+1}(t_{k-1})}{\partial AT(t_{k-1})} \cdot \frac{DT^2}{2} \qquad i = 1,3$$

and finally, for the derivative with respect to AT at all other preceeding time points $t_s, s = 0,1,\ldots k-2$

$$\frac{\partial y_i(t_k)}{\partial AT(t_s)} = \frac{\partial y_i(t_{k-1})}{\partial AT(t_s)} + \sum_{j=1}^{5} \frac{\partial f_i(t_{k-1})}{\partial y_j(t_{k-1})} \frac{\partial y_j(t_{k-1})}{\partial AT(t_s)} DT \qquad i = 2,4,5$$

10c) $$\frac{\partial y_i(t_k)}{\partial AT(t_s)} = \frac{\partial y_i(t_{k-1})}{\partial AT(t_s)} + \sum_{j=1}^{5} \frac{\partial f_i(t_{k-1})}{\partial y_j(t_{k-1})} \frac{\partial y_j(t_{k-1})}{\partial AT(t_s)} \cdot DT$$

$$+ \sum_{j=1}^{5} \frac{\partial f_{i+1}(t_{k-1})}{\partial y_j(t_{k-1})} \frac{\partial y_j(t_{k-1})}{\partial AT(t_s)} \cdot \frac{DT^2}{2}$$

with similar equations holding for the derivative with respect to TT at all time points. It is recognized that all derivatives of the state y required on the right hand side of 10) have already been formed at previous steps in the process.

This procedure continues until we reach TF and thus obtain the required derivatives of final state.

Since the cost function also depends upon TF, we are also required to form the derivative of the cost with respect to TF, however this presents no difficulty.

Results

In stating the results of using the above described computer program, it is to be noted that there were severe time limitations on this initial phase of the project so that only a minimal amount of time was left after formulation, development and checkout of the basic computer program. Consequently, the results presented herein are preliminary in the sense that no "tuning" (such as problem scaling) of the computer program to this problem was done. Such tuning will produce better and often very significantly better results than the basic program. Nevertheless, the results that were obtained indicate significant savings in time over those obtained from the presently used guidance scheme.

The basic missile target scenario that was used had the target at 20,000 feet initial range. Both missile and target had initial velocity of 800 feet per second. The missile heading and target aspect were varied as depicted by dashed lines in the figure below.

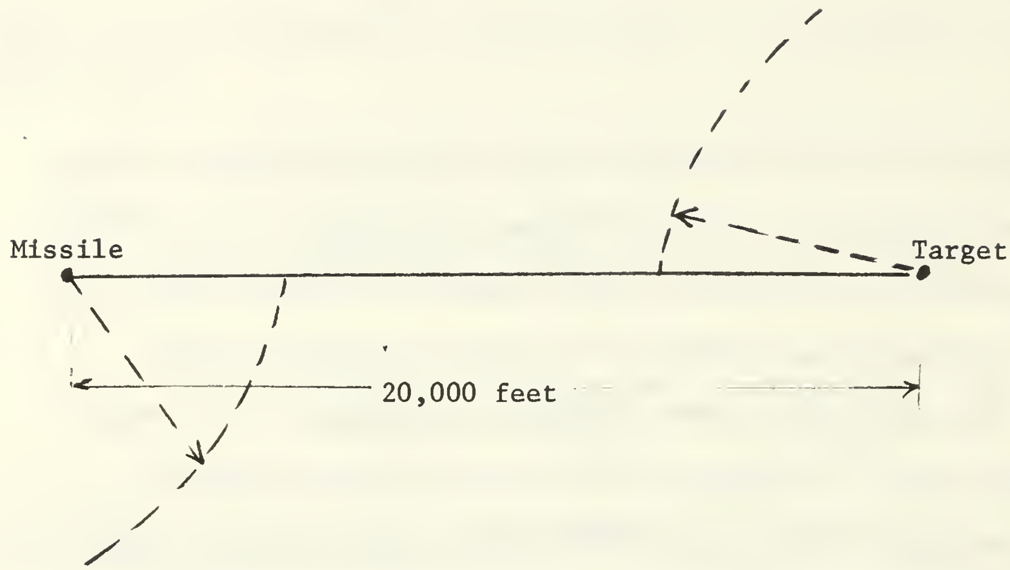


Figure 2

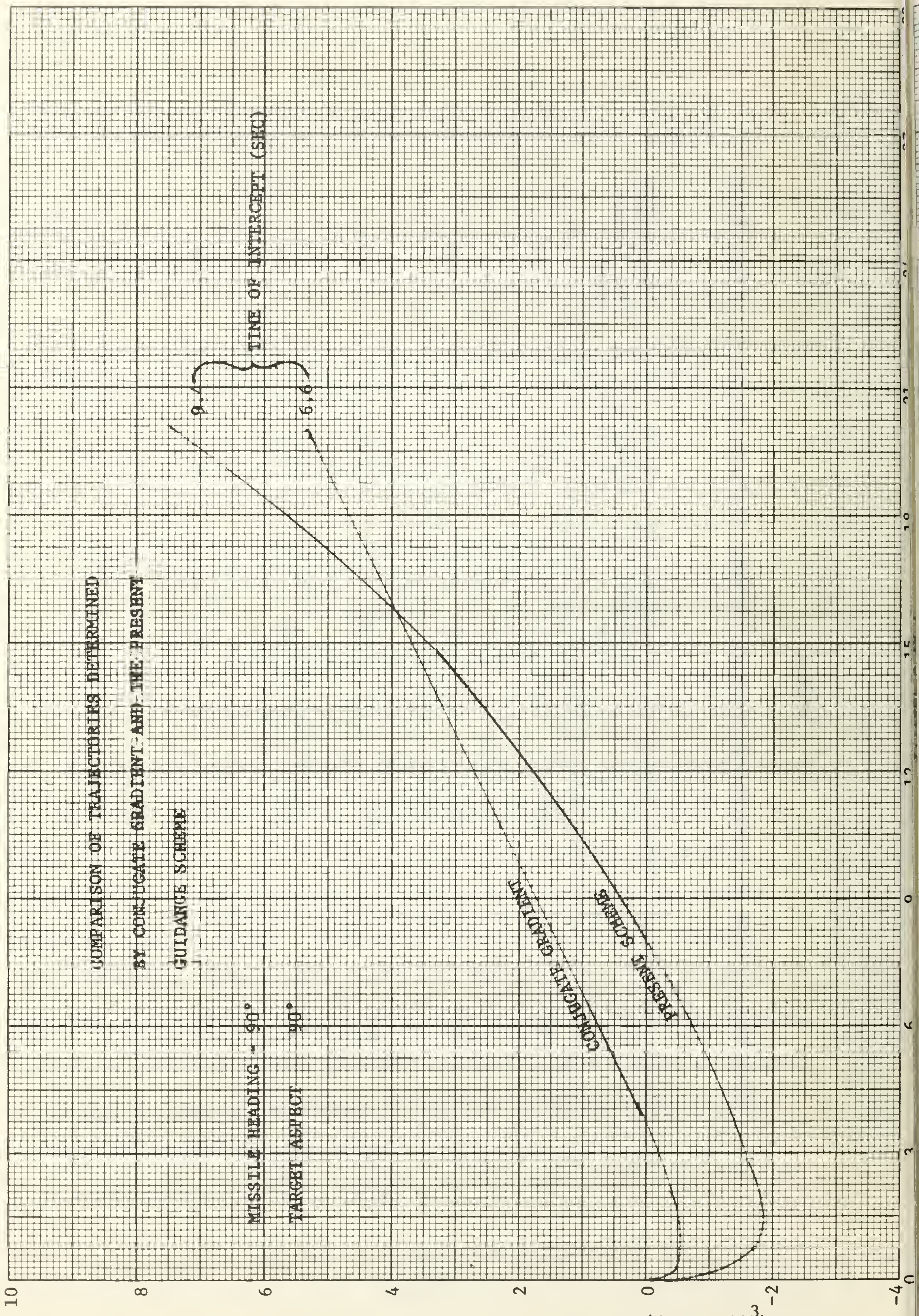
Basic Missile-Target Scenario

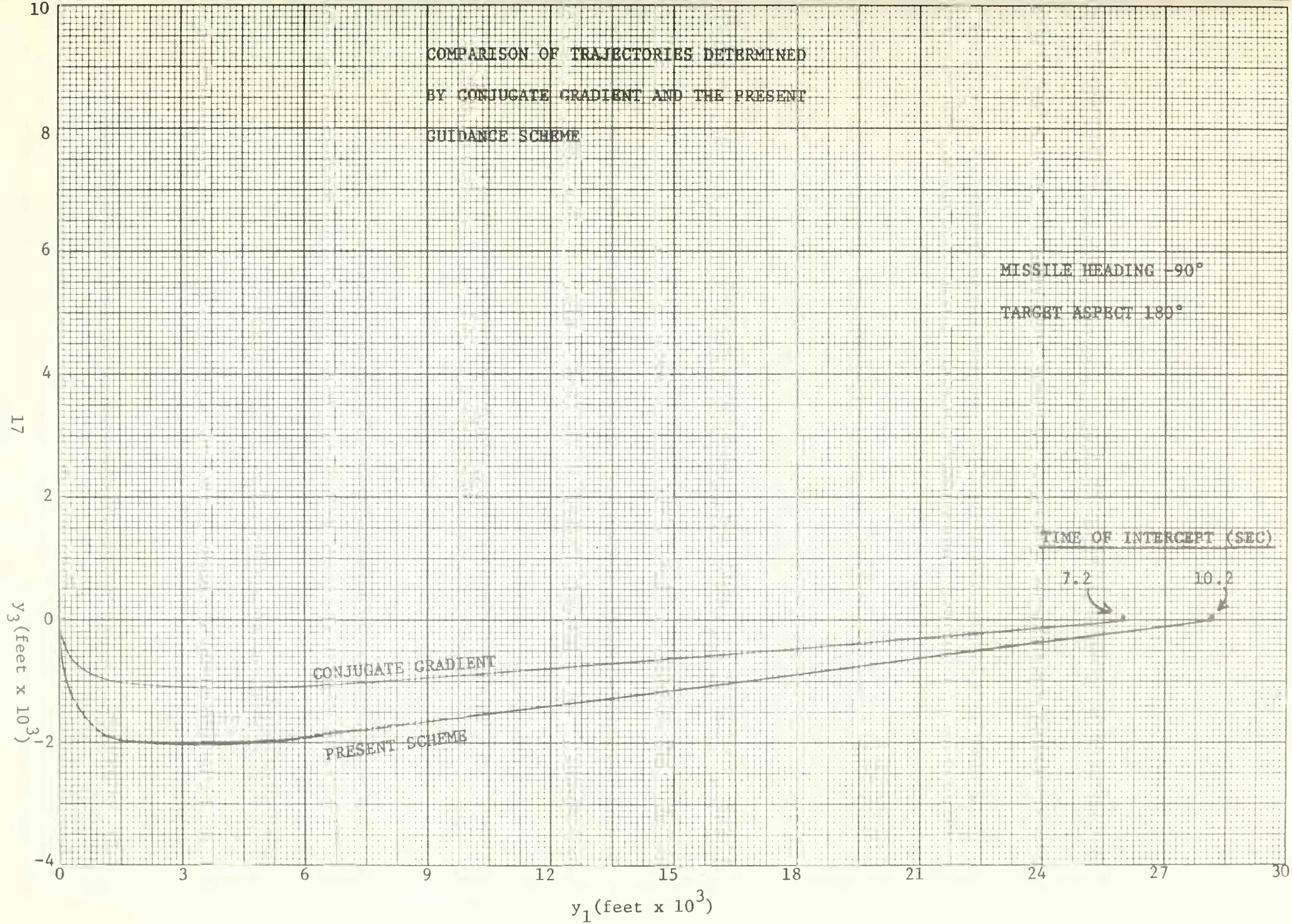
The results of the conjugate gradient runs together with a comparison to the results of the presently used guidance scheme are presented in the table which follows. In addition, plots of some of these comparison trajectories are also presented. In each plot is indicated the time of intercept with the target. Finally, the values of the control variables TT and AT at each time point t_j are listed for each plot. The number of such time points or equivalently the number of intervals in the integration process is arbitrary and was generally selected to give roughly an interval of .25 sec for the initial trajectory and time of flight which were used to start the program for each case.

Table 1

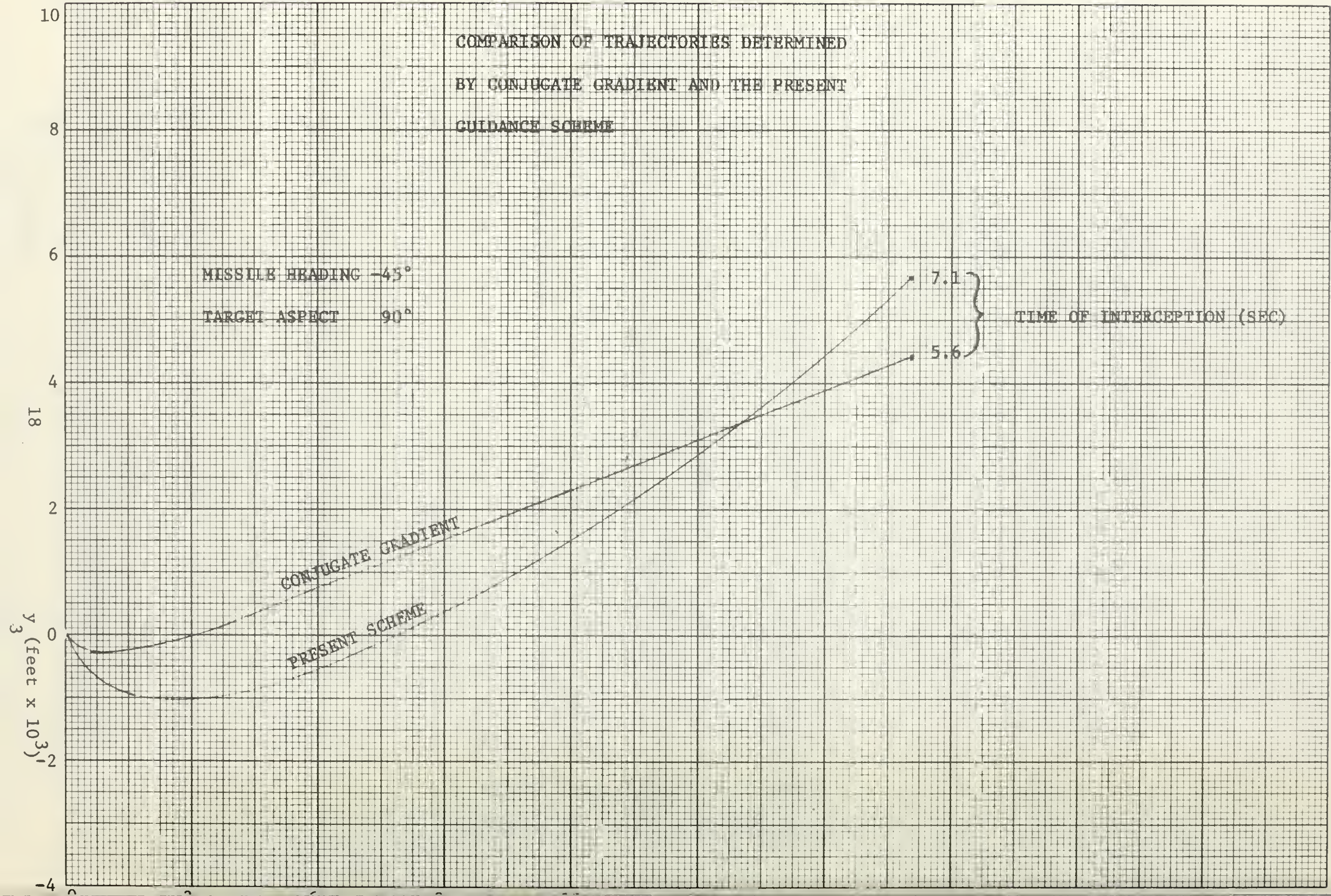
Comparison of Times to Intercept Obtained By Conjugate Gradient and Presently Used Scheme

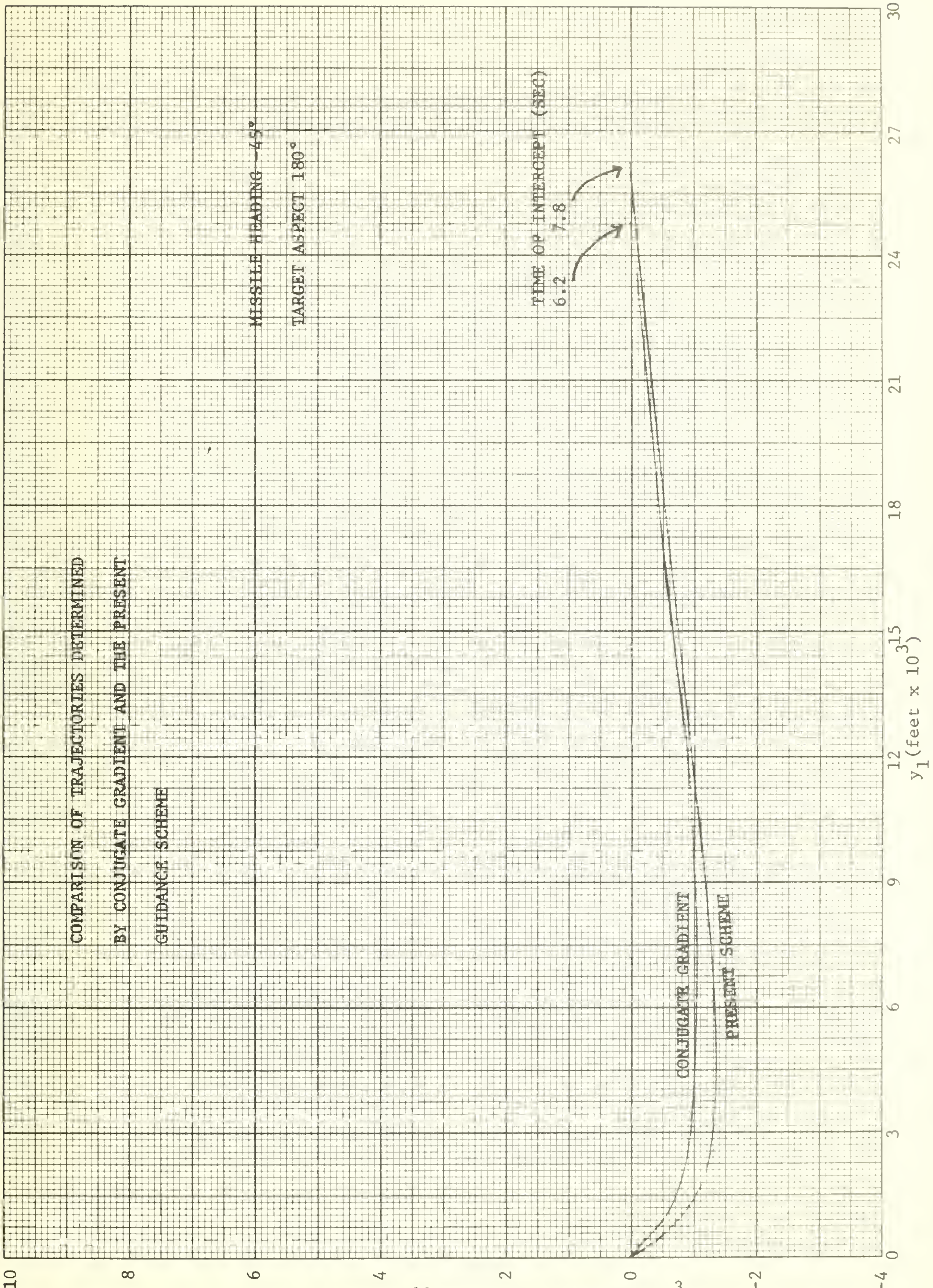
Missile Heading	Target Aspect	Conjugate Gradient Time	Time of Presently Used Scheme	% Improvement Over Present Scheme
-90°	180°	7.2	10.2	30%
-90°	90°	6.6	9.4	30%
-45°	180°	6.2	7.8	21%
-45°	90°	5.6	7.1	21%
-45°	0°	4.7	5.5	15%


COMPARISON OF TRAJECTORIES DETERMINED
BY CONJUGATE GRADIENT AND THE PRESENT
GUIDANCE SCHEME
MISSILE HEADING - 90°
TARGET ASPECT 90°
TIME OF INTERCEPT (SEC)
9.4
6.6
CONJUGATE GRADIENT
PRESENT SCHEME
10
8
6
4
2
0
-2
-4
y_3 (feet x 10^3)



COMPARISON OF TRAJECTORIES DETERMINED
BY CONJUGATE GRADIENT AND THE PRESENT
GUIDANCE SCHEME
MISSILE HEADING -90°
TARGET ASPECT 180°
TIME OF INTERCEPT (SEC)
7.2
10.2
CONJUGATE GRADIENT
PRESENT SCHEME
10
8
6
4
2
0
-2
-4
0
3
6
9
12
15
18
21
24
27
30
y1(feet x 10^3)
y3(feet x 10^3)


COMPARISON OF TRAJECTORIES DETERMINED
BY CONJUGATE GRADIENT AND THE PRESENT
GUIDANCE SCHEME
MISSILE HEADING -45°
TARGET ASPECT 90°
7.1
5.6
TIME OF INTERCEPTION (SEC)
CONJUGATE GRADIENT
PRESENT SCHEME
10
8
6
4
2
0
-2
-4
y3 (feet x 10^3)



COMPARISON OF TRAJECTORIES DETERMINED
BY CONJUGATE GRADIENT AND THE PRESENT
GUIDANCE SCHEME
MISSILE HEADING -45°
TARGET ASPECT 180°
TIME OF INTERCEPT (SEC)
6.2
7.8
CONJUGATE GRADIENT
PRESENT SCHEME
y_1 (feet x 10^3)
y_3 (feet x 10^3)

History of Thrust Magnitude (lbs.) and Direction (Radians) at Each Time Point

Missile Heading -90°
Target Aspect 90°

THRUST USED	ANGLE USED
-4.605009591450599E-06	4.7231215939[illegible]0707
14399.9999793[illegible]	.3859842903584423
14399.99997969224	.3803681117161034
14399.99998002256	.3767565214444642
14399.99998032638	.3748835541679647
14399.99998060886	.3742062955686325
14399.999980874	.3739137777928477
14399.99998117321	.3716230858999721
14399.99998147072	.3662140979628795
14399.99998178906	.3611019836883072
14399.99998213868	.3562308881011089
14399.99998[illegible]284	.351614422703192[illegible]
14399.99998298858	.3472233046921528
14399.99998352771	.3429874251066726
14399.99998417795	.3387945142194222
14399.9999849742	.3344809309495753
-1.484135400174198E-05	.3395337541740675
-1.367361742331171E-05	.3383031615855645
-1.293201954702595E-05	.3374743729385739
-1.201623031575876E-05	.3369391888427191
-1.112597441374629E-05	.33662599[illegible]40162[illegible]9
-1.[illegible]E-05	.336[illegible]57010192663
-9.421166493039155E-06	.33649044541 78[illegible]
-8.606247083233235E-06	.3366156814075165
-7.816116653991806E-06	.3368497515614064
-7.050653059250109E-06	.337185911259483
-6.309754339144073E-06	.3376216675022208
-5.593337347902797E-06	.3381581899218649
-4.901336836832315E-06	.3387990661726958
-4.233704898745038E-06	.3395529221752953
-3.590410713869574E-06	.3404271763094834
-2.971440567082996E-06	.341433874412035
-2.376798128100541E-06	.34258605604473136
-1.806505011774549E-06	.3439007378455418
-1.260601661405314E-06	.345393669010457
-7.391486353900189E-07	.3470840440900813
-2.422884241523481E-07	.34899142194[illegible]4018
0	.35000000000003[illegible]8

History of Thrust Magnitude (lbs.) and Direction (Radians) at Each Time Point

Missile Heading -90°
Target Aspect 180°

THRUST USED	ANGLE USED
-1.164666815255909E-05	4.723242903744533
14399.99998027039	6.47982294937597E-02
14399.99998047564	6.335285433950643E-02
14399.99998062214	6.43769096188719E-02
14399.9999807254	6.797136828113957E-02
14399.99998080467	7.415169752049955E-02
14399.99998095615	7.622122700236574E-02
14399.99998112485	7.700876366212161E-02
14399.99998132152	7.828509488296193E-02
14399.99998155864	8.014004523567484E-02
14399.99998185297	8.273575855923544E-02
14399.99998222689	8.632825473186248E-02
14399.99998270991	9.131522395380891E-02
14399.99998334048	9.832592525874304E-02
14399.99998416803	.1083958274495448
-1.566493161176776E-05	7.96950183133419E-02
-1.460274935475232E-05	7.59503741967636E-02
-1.357041566137291E-05	7.301455145550678E-02
-1.256871182486963E-05	7.06795697486307E-02
-1.159822829443353E-05	6.878189976333292E-02
-1.065942187043155E-05	6.719148077868842E-02
-9.752657448003086E-06	6.580201445930516E-02
-8.878238563914252E-06	6.452315951794057E-02
-8.036430022016277E-06	6.327434468444718E-02
-7.227474929841107E-06	6.197975874175827E-02
-6.451607773021927E-06	6.056412099550624E-02
-5.709064629795342E-06	5.894892171735704E-02
-5.000091267843396E-06	5.704890571965851E-02
-4.324949577890757E-06	5.476865053780268E-02
-3.683922571868313E-06	5.199917073219558E-02
-3.077043609868248E-06	4.869794646464365E-02
-2.504968690771787E-06	4.444098055129932E-02
-1.968061221118917E-06	3.92564571099527E-02
-1.470174244710638E-06	3.272654142216548E-02
-1.007127876053239E-06	2.621487103644177E-02
-5.79326339606816E-07	1.768514541362738E-02
-1.871986448563937E-07	6.513313859402577E-03
0	0

History of Thrust Magnitude (lbs.) and
Direction (Raidans) at Each Time Point

Missile Heading -45°
Target Aspect 90°

THRUST USED	ANGLE USED
-1.63353040870099E-05	4.686448382899316
14399.99995183016	.387651677708849
14399.99995216494	.3838390471804263
14399.99995247651	.3801651728985935
14399.99995275883	.3771757452566404
14399.99995308508	.3712756293081592
14399.99995338534	.3581180177472189
14399.99995369842	.3462880488682431
14399.99995403657	.3346542026327179
14399.99995441427	.3239179702043936
14399.99995484988	.3135202020455658
14399.99995536674	.3036432105026189
14399.99995599447	.2942186466090938
14399.99995677048	.2851357111224246
14399.99995774172	.2762476770389699
14399.99995896665	.2673758108020489
14399.99996051725	.2583097067560104
14399.99996248091	.2488022056680618
14399.99996496153	.2384328227279375
-3.360690567644473E-05	.2701896461683151
-3.055014234939948E-05	.2705728921977258
-2.761881665880012E-05	.2712376903123615
-2.481337271253712E-05	.2721725650935123
-2.213474462303725E-05	.2733844764567251
-1.958432548230329E-05	.2748939275923527
-1.716396195193501E-05	.2767326749345042
-1.487597041217111E-05	.2789430921864116
-1.272317324197033E-05	.2815787121560207
-1.070895599476627E-05	.2847057554868285
-8.837348332594896E-06	.2884056528652088
-7.113133978187216E-06	.2927787357020200
-5.5419[illegible]31254628E-06	.297949450815962[illegible]
-4.130724330628648E-06	.304073678922899
-2.887462000867227E-06	.3113490206533355
-1.822090320944155E-06	.3200292054864544
-9.46727132607903E-07	.3304437549857585
-2.764530322192844E-07	.3430223498022051
0	.3500000000003638

History and Thrust Magnitude (lbs.) and
Direction (Radians) at Each Time Point

Missile Heading -45°
Target Aspect 180°

THRUST USED	ANGLE USED
1.367321582005183E-05	4.837170084295014
14400.00005733645	-1.397415958209399E-02
14400.00005555382	-1.281514725891007E-02
14400.00005378942	-1.385445635105004E-02
14400.00005200076	-1.[illegible]90[illegible]1660226769E-02
14400.00005010682	-9.93658406777 94[illegible]7E-03
14400.0000481275	7.221619234648867E-04
14400.0000460542	1.034171363879729E-02
14400.00004386944	1.378848443514379E-02
14400.00004155421	2.593911288554153E-02
14400.00003908793	3.165518097112531E-02
14400.0000364487	3.574040945795267E-02
3.504169887886131E-05	7.42664882134405E-02
3.215485447067173E-05	8.1668059898296E-02
2.928264034391217E-05	8.757594125580921E-02
2.642542579467247E-05	9.238318277518468E-02
2.358333612686819E-05	9.634692870081226E-02
2.075631875359627E-05	9.962887758792461E-02
1.794418215165447E-05	.102316828755256
1.514665623229595E-05	.1044341137683111
1.236335285271146E-05	.10594002569[illegible]459
9.593842453132146E-06	.1067222295889411
6.837630698889432E-06	.1065[illegible]0729974348
4.094177052457139E-06	.1052018906808591
1.362901359636352E-06	.1021200558159281
0	9.999999999990905E-02

History of Thrust Magnitude (lbs.) and Direction

Missile Heading -45°
Target Aspect 0°

THRUST USED	ANGLE USED
-8.636091280320598E-06	5.497045771010888
14399.99999284112	4.874623557450994E-02
14399.99999235387	5.320166791474908E-02
14399.99999246009	6.068224114497995E-02
14399.99999263662	6.58399346371108F-02
14399.9999928314	7.008778557880289E-02
14399.99999305048	7.454326872394564E-02
14399.[illegible]330091	7.[illegible]401483E-02
14399.99999359178	8.452702936649252E-02
14399.99999393462	9.051504926056648E-02
14399.99999434359	9.771095047974996E-02
14399.99999483558	.1068473379757659
14399.99999542933	.1191950730603627
-4.167844072779003E-06	8.506952862118809E-02
-3.4673123333[illegible]2475E-06	7.310431814077696E-02
-2.809366742004528E-06	7.048833264681219E-02
-2.195648663283228E-06	6.185014095925982E-02
-1.628276506234843E-06	5.240253851835182E-02
-1.106310189309017E-06	4.115799875021087E-02
-6.306300486630698E-07	2.712890277855009E-02
-2.031446021637576E-07	9.850414457511216E-03
0	0

Conclusions and Recommendations

From the table, the general pattern is that the conjugate gradient trajectories have significantly shorter times to intercept for all cases with the greatest improvement occurring for the longer duration trajectories and the average improvement being around 25%. The general nature of the conjugate gradient trajectory is to burn at full throttle for as long as possible. It should be noted here that these results represent local minimums of the cost function 4) and not global minimums. There are other local minimums which may be significantly better than the ones obtained. "Tuning" of the computer program and more experimentation with our cost function, to determine its "hills and valleys," as a function of thrust magnitude and direction history will enable us to achieve these.

The purpose of the initial phase of this project has been accomplished in establishing the desirability of considering variable thrust engines in conjunction with engine gimbling to provide trajectories with significantly improved characteristics. Specifically, from these results the time to intercept has been improved, but improvement in other characteristics such as fuel used, can also be obtained. Furthermore, numerical results indicate that an engine capable only of restarting in flight rather than a continuously variable one achieves these improvements. (1)

It is noted here that this work establishes the presence of improved trajectories over the ones presently being used. Such items as mechanization of these trajectories into an actual missile have not been considered.

---

(1) However, this type of control may not provide the global minimum

Suggestions

The following extensions of this work are suggested:

a) Tuning of the computer program (problem scaling)

b) Experimentation with additional cases and with the weighting factor UN of the cost to determine the best value for reducing the time to intercept

c) Modifying the program to consider minimizing the fuel used till intercept or other trajectory parameters of interest

d) Modifying the computer program to include three dimensional trajectories.

Appendix A

Conjugate Gradient Program

In Finite Dimensional Space

```
DOUBLE PRECISION A[illegible](1),[illegible](1,1),HH(202),F
DIMENSION YO(5),AT(50),TT(50)
EXTERNAL FUNCT
            COMMON/FMAN/IFL,LF,H,NI,PY1T,PY3T,Y1TO,Y3TO,UN,YO
DATA(AT(I),I=1,38)/
DATA(TT(I),I=1,38)/
DATA N,EST,LIMIT,IFL,H,NI,PY1T,PY3T,Y1TO,Y3TO,UN, [illegible]/

DATA(YO(I),I=1,5)/
   E[illegible]=.0001
            DO 10 I=1,(N-2),2
             IN=(I+1)/2
             ARG(I)=TT(IN)
             ARG(I+1)=YT(IN)
   10       CONTINUE
             ARG(N)=TF
CALL [illegible](FUNCT,[illegible],EST,[illegible],LIMIT,IER,[illegible])
WRITE (1,111) F
111 FORMAT (1X,12HMIN COST IS=,F19.8//1X,11HMIN PTS ARE/)
DO 1415 IX=1,N
WRITE (1,1414) ARG(IX)
1414 FORMAT(1X,F19.8)
1415 CONTINUE
   END
SUBROUTINE FUNCT(N,ARG,VAL,GRAD)
DIMENSION [illegible](5),YO(5),F 1(5),YJ1(5),FYJ1(5,5),F J1(5,2),YC(5,2,50),
YJ1C(5,2,50)
DOUBLE PRECISION ARG(N)
DOUBLE PRECISION GRAD(N)
DOUBLE PRECISION VAL
DOUBLE PRECISION AMDA,T[illegible], [illegible]
DOUBLE PRECISION HH(202)
COMMON/PRINT/HH,AMBDA,IER
   C: [illegible] CLOSED [illegible] [illegible]
COMMON TAT,CAT,SAT,MN,E1,E2,CS,OSC
COMMON/FMAN/ IFL,LF,H,NI,PY1T,PY3T,Y1TO,Y3TO,UN,[illegible]
REAL MN
DANGABS=0.0
DO 1010 I=2,N-1,2
DANGMAG=DABS(ARG(I)-HH(N+I))
IF(DANGMAG.LE.DANGABS) GO TO 1010
DANGABS=DANGMAG
1010 CONTINUE
DTTABS=0.0
DO 1020 I=1,N-2,2
DTTMAG=DABS(ARG(I)-HH(N+I))
IF(DTTMAG.LE.DTTABS)GO TO 1020
DTTABS=DTTMAG
1020 CONTINUE
DISPLAY "[illegible]=",DANGABS,"DTTABS=",DTTABS,"AMBDA=",AMBDA,"IER=",
IER,"TF=",ARG(N)
             INDT=0
TPI=6.2831850
             MNI=NI+1
             IND1=0
             INDG=0
             CS=1117.77-40.92*H
OSC=.1734*.00243* EXP(-.334*H)
             DT=ARG(N)/NI
             J=2
             DO 10 I=1,5
             Y(I)=YO(I)
   10       CONTINUE
   C:
   C:
   C: BIG LOOP FOR INTEG. & DIFF. IF INTEG.
```

```
            DO 15 I=1,5
            DO 15 J=1,2
            YU(I,J,1)=0.0
     15     CONTINUE
            IF (IFL.EQ.0) GO TO 500
     C:
     C:
     C: SIMPLE INTEGRATION
     500    DO 1000 J=2,[illegible]
            DO 520 L=1,(J-1)
            DO 520 I=1,5
            DO520 K=1,2
            YJ1U(I,K,L)=YU(I,K,L)
     520    CONTINUE
            DO 530 L=1,5
            YJ1(L)=Y(L)
     530    CONTINUE
            IAT=2*(J-1)
            ITT=IAT-1
 IF(ARG(IAT).GE.0.0[illegible]) GO TO 535
            ATJ1=ARG(IAT)+TPI
            GOTO 538
     535    IF(ARG(IAT).LE.TPI) GO TO 537
            ATJ1=ARG(IAT)-TPI
            GOTO 538
     537    ATJ1=ARG(IAT)
     538    TTJ1=ARG(ITT)
            MTJ1=J-1
 CALL FU(YJ1,ATJ1,TTJ1,[illegible])
 DO 540 I=1,5
 Y(I)=YJ1(I)+FK1(I)*DT
 540 CONTINUE
 Y(1)=Y(1)+LF*FK1(2)*DT*DT/2.0
 Y(3)=Y(3)+LF*FK1(4)*DT*DT/2.0
 CALL GRADIENT(YJ1,FK1,FYJ1,FUJ1,TTJ1)
            DO 550 I=1,5
            DO 550 K=1,5
 FYJ1(I,K)=FYJ1(I,K)*DT
     550    CONTINUE
            DO 553 I=1,5
            DO 553 K=1,2
            FUJ1(I,K)=FUJ1(I,K)*DT
     553    CONTINUE
     C:
     C:
     C: [illegible] INTEG. PARTIALS
            DO 600 K=1,J
     555    IF(K.NE.J) GOTO 570
            DO 560 I=1,5
            DO560 L=1,2
 YU(I,L,J)=0.0
     560    CONTINUE
            GO TO 600
     570    IF(K.[illegible].(J-1)) [illegible]
            DO 580 I=1,5
            DO 580 L=1,2
            YU(I,L,(J-1))=FUJ1(I,L)
     580    CONTINUE
 DO 585 L=1,2
 YU(1,L,(J-1))=YU(1,L,(J-1))+LF*FUJ1(2,L)*DT/2.0
 YU(3,L,(J-1))=YU(3,L,(J-1))+LF*FUJ1(4,L)*DT/2.0
 585 CONTINUE
            GOTO 600
     590    DO 595 I=1,5
            DO 595 L=1,2
            YU(I,L,K)=YJ1U(I,L,K)
```

```
              DO 595 IJ=1,5
              YU(I,L,K)=YU(I,L,K)+FYJ1(I,IJ)*YJ1U(IJ,L,K)
   595        CONTINUE
DO 597 L=1,2
DO 597 I=1,5
YU(1,L,K)=YU(1,L,K)+LF*FYJ1(2,I)*YJ1U(I,L,K)*DT/2.0
YU(3,L,K)=YU(3,L,K)+LF*FYJ1(4,I)*YJ1U(I,L,K)*DT/2.0
597 CONTINUE
   600        CONTINUE
   1000       CONTINUE
   C: SETTING UP TARGET COORD.
              Y1T=Y1T0+DY1T*ARG(N)
              Y3T=Y3T0+DY3T*ARG(N)
              VAL=ARG(N)+UN*((Y(1)-Y1T)**2+(Y(3)-Y3T)**2)
DISTAN=(Y(1)-Y1T)**2+(Y(3)-Y3T)**2
DISPLAY"DISTAN=",DISTAN
   C:
   C:
   C: COMPUTE PARTIAL OF COST W.R.T. TF
GRAD(N)=1.0+2.0*UN*((Y(1)-Y1T)*(FK1(1)-DY1T)+(Y(3)-Y3T)*(FK1(3)-
D 3T))
   C: FORMING PARTIALS OF COSR W.R.T. U
                CTY1=2.0*UN*(Y(1)-Y1T)
              CTY3=2.0*UN*(Y(3)-Y3T)
              DO 620 K=1,(N-2),2
KK=(K+1)/2
              GRAD(K)=CTY1*YU(1,1,KK)+CTY3*YU(3,1,KK)
               GRAD(K+1)=CTY1*YU(1,2,KK)+CTY3*YU(3,2,KK)
   620        CONTINUE
   C:
   C:
   C: PRINT COST, G VIOLATIONS, BAD TAT VALUES
WRITE (1,777) VAL,INDG,IND1
   777        FORMAT(1X,4HVAL=,F19.8,5X,5HINDG=,I8,5X,5HIND1=,I8)
   C:
   C:
   C: COMPUTE FUEL USED
              FS=0.0
              DO 630 I=1,(N-2),2
              FS=FS+ARG(I)*DT
   630        CONTINUE
WRITE (1,888) FS
   888        FORMAT(1X,10HFUEL USED=,F19.8)
   C:
              C:
   C: COMPUTE THRUST VIOLATIONS AND MAX,MIN VALUES
TTMAX=14400.0D0
TTMIN=0.0D0
DO 680 I=1,(N-2),2
IF(ARG(I).GT.TTMIN) GO TO 660
TTMIN=ARG(I)
INDT=INDT+1
GO TO 680
660 IF(ARG(I).LT.TTMAX) GO TO 680
TTMAX=ARG(I)
INDT=INDT+1
680 CONTINUE
C: PRINT NUMBER OF THRUST VIOLATIONS AND MAX,MIN VALUES
WRITE (1,9999) INDT,TTMAX,TTMIN
9999 FORMAT(1X,28HNUMBER OF THRUST VIOLATIONS=,I8,/,1X,
6HTTMAX=,F19.8,/,1X,6HTTMIN=,F19.8)
1212 CONTINUE
2222 FORMAT (1X,4HVAL=,F19.8,3X//1X,10HYU(I,1,1)=,
3F19.8/1X,2F19.8/1X,10HYU(I,2,1)=,3F19.8/1X,2F19.8/1X,10HYU(I,1,2)=,
3F19.8/1X,2F19.8)
```

```
2323 FORMAT (1X,10HYU(I,1,1)=,3F1[illegible]/1X,2F1[illegible].8/1X,10HYU(I,2,1)=,
3F19.8/1X,2F19.8/1X,10HYU(I,2,2)=,3F19.8/1X,2F19.8//1X,11H[illegible])
WRITE(1,4444) CTY1,CTY3,(Y(I),I=1,5),[illegible]
4444 FORMAT(1X,5HCTY1=,F19.8,5X,[illegible]=,F1[illegible]/1X,[illegible](I)=,
3F19.[illegible]/1X,2F19.8/1X,4HY1[illegible]=,F1[illegible],5X,4H[illegible]F19.8)
5000 RETURN
END
SUBROUTINE FU(Y,AT,TT,MT,IND1,IND6,DY)
DIMENSION Y(5),B(5),DY(5)
COMMON TAT,CAT,SAT,[illegible],B1,B2,CS,[illegible]
[illegible]
REAL MN
PI=3.14159
VMS=Y(2)**2 +Y(4)[illegible]
[illegible]=S[illegible]T(VMS)
QS=QSC*VMS
MN=V[illegible]/CS
TAU=ATAN2(Y(4),Y(2))
IF(TAU.GE.0.0) GO TO 2110
TAU=TAU+2.0*PI
2110 CONTINUE
TAT=TAU-AT
AB=ABS(TAT)
IF(AB.GT.PI) GOTO 3
ALP=AB
GOTO 5
3 ALP = 2.[illegible]*PI - A[illegible]
[illegible]
[illegible]
[illegible]
[illegible] TO 12
10 IND1=IND1+1
12 IF(TAT.GE.0.0.AND.TAT.[illegible]) [illegible]
[illegible]
B1=SIN(AT)
B2=-COS(AT)
GOTO 20
15 B1=-SIN(AT)
B2=COS(AT)
[illegible]
[illegible]
C: FORMING CA AND CN FUNCTIONS
20      CCA=0.2283*COS(5.5714*ALP)
CA=-0.5565*CCA*[illegible]1(0.65+0.1[illegible],1.[illegible]7-[illegible])
CCN=-[illegible].23[illegible]7[illegible]-[illegible]*(1.0-22.[illegible]195[illegible]ALP*(1.[illegible]+[illegible]ALP*(1.0-.6656*ALP*(
1.[illegible]-.1580*ALP))))
CN=CCN+AMIN1(.30+.[illegible],1.[illegible]-.[illegible]74[illegible])
FN=CN*QS
FA=CA*QS
IF(FN/Y(5).LE.1353.0) GOT[illegible]
IND6=IND6+1
23 CAT=COS(AT)
SAT=SIN(AT)
DY(1)=Y(2)
DY(2)=((TT-FA)*CAT-FN*B1)/Y(5)
DY(3)=Y(4)
DY(4)=((TT-FA)*SAT-FN*B2)/Y(5)
DY(5)=-TT/8050.0
RETURN
END
[illegible]
SUBROUTINE GRADIENT(Y,[illegible],DFY,[illegible])
DIMENSION Y(5),DFY(5,5),DF[illegible](5,2),[illegible](5)
[illegible]
DATA PI/3.14159/
REAL [illegible]Y2,[illegible]Y4,[illegible]
```

```
      IF ([illegible] .GE. 0.0 .AND. TAT .LE. PI) GOTO 35
      IF (TAT .LE. -PI .OR. TAT .EQ. 2.0*PI) GOTO 35
      E1AT=CAT
      E2AT=SAT
      ALPTAN=-1.0
      GOTO 40
   35 ALPTAN=1.0
      E1AT=-CAT
      E2AT=-SAT
   40 ALPAT=-ALPTAN
      OSY2=OSC*2.0*Y(2)
      OSY4=OSC*2.0*Y(4)
      [illegible]=(Y(2)**2 +Y(4)**2)
      [illegible]
      [illegible]
      CT[illegible]=Y(2)/V[illegible]
      STAN=Y(4)/V[illegible]
      ALPY2=-ALP[illegible]*Y(4)/
      [illegible]ALPTAN*Y(2)/V[illegible]
      [illegible]=C[illegible]/C
      [illegible]
C
C     [illegible]ING DERIVATIVES OF DY [illegible]
      [illegible](161)
      IF ([illegible].T.PI) GO TO 43
      AL[illegible]
      [illegible] 44
   43 AL[illegible]=2.0*PI-A[illegible]
   44 CCA=0.[illegible]283*COS(2.5714*ALP)
      CA=-0.5565+CCA*AMIN1(0.[illegible]5+[illegible].13[illegible],1.1[illegible]7-[illegible])
      [illegible]=-(1.0-[illegible]*AL[illegible]*(1.0+5.9[illegible]*ALP*(1.0-.665[illegible]*ALP*(1.0-.15[illegible]
     ))))*5.2237E-2
      CN=CCN*AMIN1(.2[illegible]+0.55*[illegible],1.0[illegible]-0.[illegible]74[illegible])
      CCA[illegible]=-.387031*SIN(2.5714*AL[illegible])*AMIN1(0.[illegible]5+0.1[illegible],1.1[illegible]7-[illegible])
      IF ([illegible].LE.1.155) GO TO 45
      CA[illegible]=-.170*CCA
      GO TO 47
   45 CA[illegible]=.13*CCA
   47 [illegible]=-.[illegible]*ALP+9[illegible]1.77[illegible]4*AL[illegible]*ALP-
     2[illegible].[illegible]32836*ALP*ALP*ALP)
      CNALP=CCNALP*AMIN1(.3+.55*[illegible],1.095-0.074*MN)
      IF(MN.LE.1.275) GO TO 48
      CN[illegible]=-0.074*CCN
      GO TO 49
   48 C[illegible]=.55 *CCN
   49 [illegible]
      [illegible]
      [illegible]
      [illegible]LT[illegible]-[illegible]*CAT+C[illegible]
     )*[illegible]4)*OS-(CA[illegible])*[illegible]4)/[illegible]
      [illegible]
      [illegible](4,2)=((-(CAALP*[illegible])+[illegible])[illegible]-(CA[illegible]*SAT+CN[illegible]*E
     )*[illegible]Y2)*OS-(CA*SAT+CN*E2)*OSY2)/Y(5)
      DFY(4,4)=((-(CAAL[illegible]+C[illegible]LP*E2)*AL[illegible]Y4-(CA[illegible]*SAT+CN[illegible]*E2
     )*[illegible]Y4)*OS-(CA*SAT+CN*E2)*OSY4)/Y(5)
      DFY(4,5)=-DY(4)/Y(5)
C
C
      DFY(1,1)=0.0
      DFY(1,2)=1.0
      DFY(1,3)=0.0
      DFY(1,4)=0.0
      DFY(1,5)=0.0
      DFY(2,1)=0.0
      DFY(2,3)=0.0
      DFY(3,1)=0.0
      DFY(3,[illegible])=0.0
```

```
DFY(3,3)=0.0
DFY(3,4)=1.0
DFY(3,5)=0.0
DFY(4,1)=0.0
DFY(4,3)=0.0
DFY(5,1)=0.0
DFY(5,2)=0.0
DFY(5,3)=0.
DFY(5,4)=0.0
DFY(5,5)=0.0
DFU(2,2)=(-(CA [illegible] *CAT +C [illegible] ) [illegible]
SAT)*OS/Y(5)
DFU(4,2)=(-( [illegible]
DFU(2,2)=DFU(2,2)-T [illegible] /Y(5)
DFU(4,2)=DFU(4,2)+TT*CAT/Y(5)
DFU(1,1)=0.0
DFU(1,2)=0.0
DFU(3,1)=0.0
DFU(5,2)=0.0
DFU(3,2)=0.0
DFU(5,1)=1.0/ [illegible]
[illegible]
[illegible]
[illegible]
END
      SUBROUTINE [illegible]
      DIMENSION X(N)
      DOUBLE PRECISION [illegible]
      DIMENSION G(N)
      DIMENSION H(N)
      DOUBLE PRECISION X,G, [illegible] FX,FY,OLDF,OLDG, [illegible]
ALFA,FALFS,T,Z,W,DX,DY
      COMMON/PRINT/HH,AFF [illegible]
      DISPLAY [illegible]
      CALL FUNCT(N,X,F,G)
      [illegible] =IFF
      KOUNT=0
      IF [illegible] =0
      [illegible] +1
    1 DO 43 II=1, [illegible]
      KOUNT=KOUNT+1
      OLDF=F
      GNRM=0.D0
      DO [illegible] J=1,N
      GNRM=GNRM+G(J)*G(J)
      IF(GNRM)46,46,3
    3 IF(II-1)4,4,6
    4 DO 5 J=1,N
    5 H(J)=-G(J)
      GO TO 8
    6 A [illegible]
      DO 7 J=1,N
    7 H(J)=AMBDA*H(J)-G(J)
    8 [illegible] =0.D0
      HNRM=0.D0
      DO [illegible] =1,N
      K=J+N
      X(K)=X(J)
      [illegible] =HN [illegible] (J))
      [illegible] (J) G(J)
      [illegible] =1, [illegible]
      [illegible] (I)= [illegible]
      [illegible] CONTINUE
      IF(DY)10, [illegible]
   10 [illegible] =1. [illegible]
      [illegible]
```

```
[illegible]
[illegible]
[illegible]
      IF(ALFA-AMBDA)12,13,13
   12 AMBDA=ALFA
   13 ALFA=0.D0
   14 FX=FY
      DX=DY
      DO 15 I=1,N
   15 X(I)=X(I)+AMBDA*H(I)
      DISPLAY "OLDF=",OLDF,"NOW..."
      DISPLAY "THRUST USED",     "ANGLE USED"
      DO 800 II=1,N-1,2
      DISPLAY X(II), X(II+1)
[illegible]
      CALL FUNCT(N,X,F,G)
[illegible]
[illegible]
[illegible]
      DO 16 I=1,N
   16 DY=DY+G(I)*H(I)
      IF(DY)17,20,20
   17 IF(FY-FX)18,20,20
   18 AMBDA=AMBDA+ALFA
      ALFA=AMBDA
      IF(HNRM*AMBDA-1.D10)14,14,19
   19 IER=2
      RETURN
   20 T=0.
   21 IF(...)...
      ...*(FX-FY)/AMBDA+...
      ...(...),...(...),DABS(DY))
      DALFA=.../ALFA
      DALFA=...
      IF(DALFA)23,27,27
   23 DO 24 J=1,N
      K=N+J
   24 X(J)=H(K)
      DISPLAY "OLDF=",OLDF,"..."
      CALL FUNCT(N,X,F,G)
      IER=IER
   25 IF(IER)47,26,47
   26 IER=-1
      GOTO 1
   27 W=ALFA*DSQRT(DALFA)
      ALFA=(DY+W-Z)*AMBDA/(DY+2.D0*W-DX)
      DO 28 I=1,N
   28 X(I)=X(I)+(T-ALFA)*H(I)
      DISPLAY "OLDF=",OLDF,"NOWF=",F
      CALL FUNCT(N,X,F,G)
      I...=IER
      IF(F-FX)29,29,30
   29 IF(F-FY)32,32,30
   30 DALFA=0.D0
      DO 31 I=1,N
   31 DALFA=DALFA+G(I)*H(I)
      IF(DALFA)3...,3...,3...
      IF(F-...)3...,...,...
      ...(T)-DALFA)34,32,34
   32 FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 21
   3... IF(FY-F)37,36,37
      IF(T...-...ALFA)...,...,...
```

```
          DY=F
          DY=DALFA
          AXHIA=ORIGA-ALFA
          GO TO 20
      38  T=0.D0
          DO 39 J=1,N
          J=J+1
          H(J)=Y(J)-X(J)
      39  T=T+DABS(H(J))
      DO 735 I=1,2*N
      HP(I)=H(I)
      735 CONTINUE
          IF(KOUNT-N1)41,40,40
      40  IF(T-EPS)45,45,41
      41  IF(OLDF-F+EPS)10,25,42
      42  OLDG=GNRM
          IF(KOUNT-LIMIT)43,44,44
   43  IER=0
          GO TO 1
      44  IER=1
          IF(GNRM-EPS)46,46,47
      45  IF(GNRM-EPS)47,46,47
      46  IER=
      47  RETURN
          END
```

Appendix B

Conjugate Gradient Program

In Infinite Dimensional Space

```
C: THIS PROG[illegible] DCE[illegible] CON[illegible]. GRA[illegible]. FO[illegible] [illegible]ISSLE T[illegible].



DIMENSION TT(81),AT(81),E1(81),E2(81),B(5),Y(5),[illegible](5),TAT(81),
Y2(81),Y4(81),CA(81),[illegible](81),CAT(81),SAT(81),Y5(81),[illegible],[illegible](81),
[illegible](5),[illegible](5),HAT(81),HT(81),S1(81),S2(81),T11(81),[illegible](81),Y12(81)
DIMENSION Y14(81),Y1(5),YO(5),[illegible](81),[illegible](81),[illegible](81),[illegible],
CA1(81),CAT1(81),SAT1(81),Y15(81),[illegible],[illegible](81),[illegible](81),ATL(81),
TTL(81),EL1(81),EL2(81),TATI(81),[illegible](81),[illegible](81),[illegible](81)
DIMENSION CNL(81),CATL(81),SATL(81),[illegible](81),[illegible](81),[illegible]
DATA ITMAX,ITGMAX,[illegible]/
DATA NC0,NC1,NC2,NC3,NC4,NC5,NC6,NC7,[illegible],NC9,NC10,NC11/.[illegible]1,
3.248[illegible]197,29.609739,-20.952[illegible]79,4.1362[illegible],-.1[illegible],.50764409,
-.12286171,1.35768[illegible],-1.[illegible]471,.3527[illegible],-.[illegible]3/
DATA (TT(I),I=1,81)/


DATA (AT(I),I=1,14)/


DATA (AT(I),I=15,25)/


DATA ([illegible](I),I=1,[illegible])/
DATA [illegible],TE,[illegible]/

DATA (B(I),I=1,5)/0.0,3.[illegible],-[illegible],[illegible],-6.[illegible]18/
DATA CC0,CC1,CC2,CC3,CC4,CC5,CC6,CC7,[illegible],CC9,CC10,CC11/.3[illegible],
2.537371,-11.9[illegible]72,11.09[illegible]11,-[illegible].75[illegible],.[illegible],.7[illegible],
.3533[illegible],-.25[illegible],.0711[illegible]1[illegible],-.[illegible]16,.0017203[illegible]/
PI=3.14159
2 ITG=1
IND1=0
INDG=0
ITL=0


C: COMPUTE INITIAL GRADIENT TRA[illegible]Y
DO 4 I=1,5
Y(I)=YO(I)
4 CONTINUE
CS=1117.77-40.92*H
QSC=.1734*.00243*EX[illegible](-.3[illegible]/[illegible])
[illegible]
[illegible] I=1,[illegible]1
Y[illegible]=Y(2)
Y4([illegible]T)=Y([illegible])
[illegible]*2.0
VM=SQRT(V[illegible]S)
QS=QSC*V[illegible]S
MN=VM/CS
TAU=ATAN2([illegible]([illegible]),[illegible]([illegible]))
TAT([illegible]T)=TA[illegible]-AT([illegible])
AB=ABS(TAT([illegible]T))
IF(AB.GT.PI) GOTO 3
ALP=AB
GOTO 5
3 ALP = 2.0 * PI - AB
5 DO [illegible] I=1,5
IF(T[illegible]([illegible]T).EQ.[illegible](I)) [illegible]
```

```
  CONTINUE
[illegible]
[illegible]
[illegible]
[illegible]
E2(MT)=-COS(AT(MT))
GOTO 20
15 E1(MT)=-SIN(AT(MT))
E2(MT)=COS(AT(MT))


C: FORMING CA AND CN FUNCTIONS
20 ALP2=ALP*ALP
ALP3=ALP2*ALP
ALP4=ALP3*ALP
ALP5=ALP4*ALP
[illegible]
[illegible]
[illegible]
[illegible]
C1=CC0+CC1*ALP+CC2*ALP2+CC3*ALP3+CC4*ALP4+CC5*ALP5
C2=CC6+CC7*MN+CC8*MN2+CC9*MN3+CC10*MN4+CC11*MN5
CA(MT)=C1*C2
N1=NC0+NC1*ALP+NC2*ALP2+NC3*ALP3+NC4*ALP4+NC5*ALP5
N2=NC6+NC7*MN+NC8*MN2+NC9*MN3+NC10*MN4+NC11*MN5
CN(MT)=N1*N2
FN=CN(MT)*QS
FA=CA(MT)*QS
IF(FN/Y(5).LE.1353.0) GOTO 23
INFG=INFG+1
23 CAT(MT)=COS(AT(MT))
SAT(MT)=SIN(AT(MT))
YS(MT)=Y(5)
DY(1)=Y(2)
DY(2)=((TT(MT)-FA)*CAT(MT)-FN*E1(MT))/Y(5)
F2(MT)=DY(2)
DY(3)=Y(4)
DY(4)=((TT(MT)-FA)*SAT(MT)-FN*E2(MT))/Y(5)
F4(MT)=DY(4)
DY(5)=-TT(MT)/8050.0
IF ( [illegible] .EQ.81) GOTO 30
C: SIMPLE INTEGRATION
Y(1)=Y(1)+(DY(1)+DY(2)*DT/2.0)*DT
Y(2)=Y(2)+DY(2)*DT
Y(3)=Y(3)+(DY(3)+DY(4)*DT/2.0)*DT
Y(4)=Y(4)+DY(4)*DT
Y(5)=Y(5)+DY(5)*DT
30 CONTINUE


C: SETTING TARGET COORD.
Y1T=Y1T0+DY1T*TF
Y3T=Y3T0+DY3T*TF


C: COST EXPRESSION
CT=TF+ON*((Y(1)-Y1T)**2.0+(Y(3)-Y3T)**2.0)


C: SET Y AND DY VALUES FOR CUF COMPUTATION
Y1F=Y(1)
Y3F=Y(3)
DY1F=DY(1)
DY3F=DY(3)
[illegible]
32 WRITE (1,501) ITG,[illegible]
```

```
 501 FORMAT(4HITG=,I8,5HIND1=,I8,5HINMG=,I8,//,4HITL=,I8,[illegible])
C:FORMING LAMBDA,HT,HAT,CTTF
C: DERIVATIVE OF COST W.R.T. FINAL TIME
CTTF= 1.0 + 2.0 *UN*((Y1F-Y1T) * (DY1F-DY1T) + (Y3F-Y3T)*(DY3F-
DY3T))
C: SETTING FINAL VALUES OF LAMBDA
LAM(1)= 2.0 *UN* (Y1F-Y1T)
LAM(2)=0.0
LAM(3)=2.0*UN*(Y3F-Y3T)
LAM(4)=0.0
LAM(5)=0.0
C: LOOP FOR GETTING GRADIENT AND LAMBDA
DT=TF/80.0
DO 60 MT=81,1
C: EQN. FOR GETTING DERIVATIVES W.R.T. STATE
IF (TAT(MT) .GE. 0.0 .AND. TAT(MT) .LE. PI) GOTO 35
IF (TAT(MT) .LE. -PI .OR. TAT(MT) .EQ. 2.0*PI) GOTO 35
E1AT=CAT(MT)
E2AT=SAT(MT)
ALPTAU=-1.0
GOTO 40
 35 ALPTAU=1.0
E1AT=-CAT(MT)
E2AT=-SAT(MT)
 40 ALPAT=-ALPTAU
QSY2=QSC*2.0*Y2(MT)
QSY4=QSC*2.0*Y4(MT)
VMS=(Y2(MT)**2.0+Y4(MT)**2.0)
VM=SQRT(VMS)
QS=QSC*VMS
CTAU=Y2(MT)/VM
STAU=Y4(MT)/VM
ALPY2=-ALPTAU*CTAU*[illegible]Y4(MT) / (Y2(MT)*Y2(MT))
ALPY4=ALPTAU*CTAU**2/Y2(MT)
MNY2=CTAU/CS
MNY4= STAU/CS
C: [illegible]
[illegible]
IF ([illegible]) [illegible]
ALP=AB
GO TO 44
 43 ALP=2.0*PI-AB
 44 ALP2=ALP*ALP
ALP3=ALP2*ALP
ALP4=ALP3*ALP
ALP5=ALP4*ALP
[illegible]
[illegible]
[illegible]
MN5=MN4*MN
C1ALP=CC1+2.0*CC2*ALP+3.0*CC3*ALP2+4.0*CC4*ALP3+5.0*CC5*ALP4
C2MN=CC7+2.0*CC8*MN+3.0*CC9*MN2+4.0*CC10*MN3+5.0*CC11*MN4
N1ALP=NC1+2.0*NC2*ALP+3.0*NC3*ALP2+4.0*NC4*ALP3+5.0*NC5*ALP4
N2MN=NC7+2.0*NC8*MN+3.0*NC9*MN2+4.0*NC10*MN3+5.0*NC11*MN4
C1=CC0+CC1*ALP+CC2*ALP2+CC3*ALP3+CC4*ALP4+CC5*ALP5
C2=CC6+CC7*MN+CC8*MN2+CC9*MN3+CC10*MN4+CC11*MN5
N1=NC0+NC1*ALP+NC2*ALP2+NC3*ALP3+NC4*ALP4+NC5*ALP5
N2=NC6+NC7*MN+NC8*MN2+NC9*MN3+NC10*MN4+NC11*MN5
```

```
CAALP=C2*C1ALP
CAMN=C1*C2MN
CNALP=N2*N1ALP
CNMN=N1*N2MN
F2Y2=((-(CAALP*CAT(MT)+CNALP*E1(MT))*ALPY2-(CAMN*CAT(MT)+CNMN*E1(MT)
)*MNY2)*QS-(CA(MT)*CAT(MT)+CN(MT)*E1(MT))*QSY2)/Y5(MT)
F2Y4=((-(CAALP*CAT(MT)+CNALP*E1(MT))*ALPY4-(CAMN*CAT(MT)+CNMN*E1(MT)
)*MNY4)*QS-(CA(MT)*CAT(MT)+CN(MT)*E1(MT))*QSY4)/Y5(MT)
F2Y5=-F2(MT)/Y5(MT)
F4Y2=((-(CAALP*SAT(MT)+CNALP*E2(MT))*ALPY2-(CAMN*SAT(MT)+CNMN*E2(MT)
)*MNY2)*QS-(CA(MT)*SAT(MT)+CN(MT)*E2(MT))*QSY2)/Y5(MT)
F4Y4=((-(CAALP*SAT(MT)+CNALP*E2(MT))*ALPY4-(CAMN*SAT(MT)+CNMN*E2(MT)
)*MNY4)*QS-(CA(MT)*SAT(MT)+CN(MT)*E2(MT))*QSY4)/Y5(MT)
F4Y5=-F4(MT)/Y5(MT)


C: FORMING D.E. FOR LAMBDA
DLAM(1)=0.0
DLAM(2)=LAM(1)+LAM(2)*F2Y2+LAM(4)*F4Y2
DLAM(3)=0.0
DLAM(4)=LAM(2)*F2Y4+LAM(3)+LAM(4)*F4Y4
DLAM(5)= LAM(2) * F2Y5 + LAM(4) * F4Y5


C: DERIVATIVES OF DY W.R.T. THETA
F2T=1.0/Y5(MT)
F4T=1.0/Y5(MT)
F5T=-1.0/F(50.0
F2AT=(-(CAALP*CAT(MT)+CNALP*E1(MT))*ALPT-CN(MT)*E1AT+CA(MT)*SAT(MT)
)*QS/Y5(MT)
F4AT=(-(CAALP*SAT(MT)+CNALP*E2(MT))*ALPT-CN(MT)*E2AT-CA(MT)*CAT(MT)
)*QS/Y5(MT)


C: SETTING INSTANTANEOUS GRADIENT
HT(MT)= LAM(2) * F2T +LAM(4) * F4T + LAM(5) * F5T
HAT(MT)= LAM(2) *F2AT + LAM(4) * F4AT
IF (MT.EQ.1) GO TO 60


C: SIMPLE INTEGRATION
LAM(2) = LAM(2) + DLAM(2) * DT
LAM(4) = LAM(4) + DLAM(4) * DT
LAM(5) = LAM(5) + DLAM(5) * DT
60 CONTINUE


C: SETTING UP INITIAL ITERATION ALONG SEARCH DIRECTION
IF (ITG.NE.1) GO TO 70
S3=0.0
BETA=0.0
BD=0.0
DO 65 MT=1,81
S1(MT)=0.0
S2(MT)=0.0
65 CONTINUE


C: SIMPLE INTEGRATION FOR BN
70 BN=0.0
DO 72 J= 1,80
BN= BN + (HAT(J )*HAT(J )+HT(J )*HT(J ))*DT
72 CONTINUE
BN= BN + CTTF * CTTF
IF(ITG.EQ.1) GO TO 74
BETA=BN/BD
```

```
GOTO 76
74 DO 75 J=1,81
TTL(J)=TT(J)
ATL(J)=AT(J)
FL1(J)=F1(J)
FL2(J)=F2(J)
TAUL(J)=TAU(J)
YL2(J)=Y2(J)
YL4(J)=Y4(J)
CAL(J)=CA(J)
CNL(J)=CN(J)
CATL(J)=CAT(J)
SATL(J)=SAT(J)
YL5(J)=Y5(J)
FL3(J)=F3(J)
FL4(J)=F4(J)
75 CONTINUE
DYL1F=DY1F
DYL3F=DY3F
INP1L=INP1
IJNGL=IJNG
TFL=TF
CTL=CT
YL1F=Y1F
YL3F=Y3F
76 IT=1
77 IF(IT.NE.1) GOTO 80
STEP=STEPO
S3=-CTTF + BETA *S3
ITL=0
STEPL=0.0
DO 79 J=1,81
S1(J)=-ET(J)+BETA*S1(J)
S2(J)=-AT(J)+BETA*S2(J)
79 CONTINUE
GOTO 85
80 IF (IT.NE.ITMAX) GO TO 85
WRITE(1,600) IT,ITMAX,ITG,STEPL,CTL
600 FORMAT (4HIT= ,I8,6HITMAX=,I8,4HITG=,I8,6HSTEPL=,F19.8,4HCTL=
,F19.8)
GOTO 147
85 INDT=0
FS=0.0
TF1=TFL+STEP*S3
DO 95 J=1,81
TT1(J)=TTL(J)+STEP*S1(J)
AT1(J)=ATL(J)+STEP*S2(J)
IF(AT1(J).GE.0.0)GOTO 86
AT1(J)=AT1(J)+2.0*PI
GOTO 87
86 IF(AT1(J).LE.2.0*PI)GO TO 87
AT1(J)=AT1(J)-2.0*PI
87 IF(TT1(J).LE.14400.0)GOTO 89
TT1(J)=14400.0
INDT=INDT+1
GOTO 91
89 IF(TT1(J).GE.0.0)GOTO 91
TT1(J)=0.0
INDT=INDT+1
91 FS=FS+TT1(J)*DT
95 CONTINUE
IF (FS.LE.38500.0)GO TO 98
DO 97 J1=1,81
TT1(J1)=38500.0/FS*TT1(J1)
97 CONTINUE
WRITE(1,700) ITG,IT
```

```
700 FORMAT(6HTOO MU,6HCE F E,6HL U LU,4HITG=,I8,3HI1=,I8)
C INTEGRATE STEPPED TRAJECTORY
98 DO 105 J=1,5
Y1(J)=YO(J)
105 CONTINUE
CS=1117.77-40.92*H
QSC=0.1734*.00243*EXP(-.334/H)
DT1=TF1/80.0
INDG=0
IND1=0
DO 115 J=1,81
Y12(J)=Y1(2)
Y14(J)=Y1(4)
VMS=Y1(2)**2.0+Y1(4)**2.0
VM=SQRT(VMS)
QS=QSC*VMS
MN=VM/CS
TAU=ATAN2(Y1(4),Y1(2))
TAT1(J)=TAU-AT1(J)
AB=ABS(TAT1(J))
IF(AB.GT.PI)GOTO 117
ALP=AB
GOTO 120
117 ALP=2.0*PI-AB
120 DO 125 I=1,5
IF(TAT1(J).EQ.B(I))GOTO 130
125 CONTINUE
130 IND1=IND1+1
IF(TAT1(J).GE.0.0.AND.TAT1(J).LE.PI) GOTO 135
IF(TAT1(J).LE.-PI) GOTO 135
E11(J)=SIN(AT1(J))
E12(J)=-COS(AT1(J))
GOTO 140
135 E11(J)=-SIN(AT1(J))
E12(J)=COS(AT1(J))
C: FORMING CA1 AND CN1 FUNCTIONS
140 ALP2=ALP*ALP
ALP3=ALP2*ALP
ALP4=ALP3*ALP
ALP5=ALP4*ALP
MN2=MN*MN
MN3=MN2*MN
MN4=MN3*MN
MN5=MN4*MN
C11=CC0+CC1*ALP+CC2*ALP2+CC3*ALP3+CC4*ALP4+CC5*ALP5
C12=CC6+CC7*MN+CC8*MN2+CC9*MN3+CC10*MN4+CC11*MN5
CA1(J)=C11*C12
N11=NC0+NC1*ALP+NC2*ALP2+NC3*ALP3+NC4*ALP4+NC5*ALP5
N12=NC6+NC7*MN+NC8*MN2+NC9*MN3+MN4*NC10+NC11*MN5
CN1(J)=N11*N12
FN=CN1(J)*QS
FA=CA1(J)*QS
IF(FN/Y1(5) .LT. 1553.0)) GOTO 113
INDG=INDG +1
113CAT1(J)=COS(AT1(J))
SAT1(J)=SIN(AT1(J))
Y15(J)=Y1(5)
DY1(1)=Y1(2)
DY1(2)=((TT1(J)-FA)*CAT1(J)-FN*E11(J))/Y1(5)
F12(J)=DY1(2)
DY1(3)=Y1(4)
DY1(4)=((TT1(J) - FA) * SAT1(J) - FN * E12(J)) / Y1(5)
F14(J)=DY1(4)
```

```
DY1(5)=-TT1(J) / 8050.0
IF(J .EQ. 81) GOTO 115


C: SIMPLE INTEGRATION
Y1(1)=Y1(1)+(DY1(1)+DY1(2)*DT1/2.0)*DT1
Y1(2)=Y1(2)+DY1(2)*DT1
Y1(3)=Y1(3)+(DY1(3)+DY1(4)*DT1/2.0)*DT1
Y1(4)=Y1(4)+DY1(4)*DT1
Y1(5)= Y1(5)+DY1(5)*DT1
115 CONTINUE

C: SETTING TARGET COORDINATES
Y1T1=Y1T0+DY1T*TF1
Y3T1=Y3T0+DY3T*TF1
C:
C:
C: COM[illegible] COST
CT1= TF1 [illegible]
[illegible] (1,[illegible]) [illegible],FS,[illegible],CT1
800 FORMAT (4HITG=,I8,3HIT=,I8,5HINDT=,I8,5HINDI=,I8,//,[illegible]=,
F19.8,5HINDG=,I8,4HCT1=,F19.8)
IF(CT1 .GE. CTL) GOTO 130
DO 125 J1=1,81
TTL(J1)=TT1(J1)
ATL(J1)=AT1(J1)
EL1(J1)=E11(J1)
EL2(J1)=E12(J1)
TATL(J1)=TAT1(J1)
YL2(J1)=Y12(J1)
YL4(J1)=Y14(J1)
CAL(J1)=CA1(J1)
CNL(J1)=CN1(J1)
CATL(J1)=CAT1(J1)
SATL(J1)=SAT1(J1)
YL5(J1)=Y15(J1)
FL2(J1)=F12(J1)
FL4(J1)=F14(J1)
125 CONTINUE
INDL=IND1
INDGL=INDG
TFL=TF1
CTL=CT1
STEPL=STEP
YL1F=Y1(1)
YL3F=Y1(3)
DYL1F=DY1(1)
DYL3F=DY1(3)
ITL=IT
GOTO 140
130 STEP=STEP/2.0
IF(STEP .LT. STEPM) GOTO 145
140 IT=IT+1
GOTO 77
145 WRITE(1,900)STEP,STEPM,ITL,CTL,ITG,STEPL
900 FORMAT(5HSTEP=,F19.8,6HSTEPM=,F19.8,4HITL=,I8,//,4HCTL=,F19.8,
4HITG=,I8,5HSTEPL=,F19.8)
147 IF(CTL.EQ.CT) GOTO 2000
DO 150 J1=1,81
TT(J1)=TTL(J1)
AT(J1)=ATL(J1)
Y2(J1)=YL2(J1)
Y4(J1)=YL4(J1)
E1(J1)=EL1(J1)
E2(J1)=EL2(J1)
TAT(J1)=TATL(J1)
```

```
CA(J1)=CAL(J1)
CN(J1)=CNL(J1)
CAT(J1)=CATL(J1)
SAT(J1)=SATL(J1)
Y5(J1)=YL5(J1)
F2(J1)=FL2(J1)
F4(J1)=FL4(J1)
150 CONTINUE
TF=TFL
BD=BN
CT=CTL
IND1=IND1L
INDG=INDGL
Y1F=YL1F
Y3F=YL3F
DY1F=DYL1F
DY3F=DYL3F
IF(ITG.GT.ITMAX)GOTO 2100
ITG=ITG+1
Y1T=Y1TO+DY1T*TFL
Y3T=Y3TO+DY3T*TFL
GOTO 32
2000 WRITE(1,902)ITG
902 FORMAT(6HNO IMP,6HROVEME,6HNT POS,6HSIBLE ,6HFROM G,
6HRADIEN,6HT IN T,6HHIS DI,6HRECTIO,6HN,     ,6HITG=   ,I8)
C: (NO IMPROVEMENT POSSIBLE FROM GRADIENT IN THIS DIRECTION)
GOTO 5000
2100 WRITE(1,903)
903 FORMAT(6HITG=IT,6HGMAX   )
5000 STOP
END
```

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Some interceptor missiles as presently formulated possess a programmed thrust magnitude history with a gimbaled engine to provide steering. We examine one such missile to determine whether performance can be improved if we allow a variable thrust magnitude together with engine gimbaling to provide control.

Two trajectory optimization programs were written to provide an initial answer to this problem. Preliminary results indicate reductions in the time to intercept by as much as thirty per-cent over that obtained by the presently

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